ROBUSTNESS EVALUATION OF SOLUTIONS FOR THE CAPACITATED
ARC ROUTING PROBLEM

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ABSTRACT

This paper presents the first work on stochastic arc routing problems, asStochastic CARP which is a step to provide more profitable solutions to industrial experts taking into account variations of the quantities. The experiments based on well-known sets of benchmarks prove there is a great interest in tackling random quantities. For instance, one can hope to efficiently solve real-life problems which occur in urban waste collection.

1 INTRODUCTION TO ARC ROUTING PROBLEMS

The *Capacitated Arc Routing Problem* (*CARP*) is defined on an undirected network $G=(V,E)$ with a set V of n nodes and a set E of m edges. A fleet of vehicles of identical capacity Q is based at depot node s. A subset R of required edges must be serviced by one vehicle. All edges can be traversed several times. Each edge (i,j) has a traversal cost $c_{ij} \geq 0$ and a demand $r_{ij} \geq 0$. The CARP consists in determining vehicle trips of minimal total cost, such that each trip starts and ends at the depot node and each required edge is entirely serviced by one single vehicle, taking into account the vehicle capacity Q. The cost of a trip comprises the cost of its serviced edges and of its intermediate connecting paths. Many applications can be modelled as CARP, e.g. urban waste collection (used as example in the sequel), snow plowing, sweeping, gritting. The undirected CARP permits to take into account roads whose both sides can be serviced during one traversal and in any direction. For the directed CARP, each arc represents one street (or one side of a street) with fixed direction for service. Note the difference between an edge (i,j) and a pair of opposite arcs (i,j) and (j,i). For a pair of arcs, each side needs to be serviced separately. Both the directed and undirected CARP are NP-hard, even in the single-vehicle case named *Rural Postman Problem*.

In urban waste collection, the quantities to collect in street networks are not exactly known and are evaluated by industrial experts. In fact, these quantities are random variables and the problem becomes stochastic.

This paper focuses on the stochastic evaluation of solutions computed using a deterministic model of the problem. The remainder of this paper is organized as follows. First, the sensitivity is defined under different assumptions about the vehicles management rules, focusing on the waste collection problem. Second, a framework is proposed to obtain high-quality solutions and to evaluate the sensitivity of solutions cost and of the number of trips. The framework uses the most powerful method ever published for the CARP, the Hybrid Genetic Algorithm (HGA) of Lacomme et al. (2001a). Third, an approach is defined to obtain solutions with low sensitivity to variations in quantities. The first approach is named Tight Approach and the second one Slack Approach because the vehicles are not completely filled during the optimization process. Experiments have been performed using the benchmarks of DeArmon (DeArmon, 1981), Belenguer (Belenguer and Benavent, 1997) and Eglese (Eglese and Li, 1996).

2 STOCHASTIC CARP

2.1 SCARP definition

For the deterministic CARP the demand $r_{ij} \geq 0$ are known and fixed. For the Stochastic CARP (SCARP) the demands r_{ij} are random variables. The variations of the demand may increase the number of trips required to service the arcs. Because the number of trips is linked to the solution cost, these variations may increase in turn the solution cost. The challenging problem consists in determining solutions for which the variations of the cost are bounded or minimal. The mathematical formulation of the problem is the following:

- Ω — the set of all solutions
- x — a solution of the basic deterministic CARP (set of trips).
- $h(x)$ — the solution cost for the solution x. The demand r_{ij} is known and determinist.
- $H(x,\omega)$ — the solution cost if random events occur.
- $\overline{H(x,n)}$ — the empirical average cost over n independent evaluations of $H(x,\omega)$
- $s(H(x,\omega))$ — the empirical standard deviation

The objective of the SCARP consists in computing a solution for which the random events consequences are lower. Our aim is not to solve the Stochastic CARP but:

- to study the sensitivity of solutions computed using a deterministic model of the problem by evaluating $\overline{H(x,n)}$, $s(H(x,\omega))$;
- to provide the basic concepts of a new technique to identify more robust solutions.

2.2 Hypotheses on vehicles management rules

When a vehicle reaches is capacity, the vehicle goes from its current position in the network to the depot node and returns to its position to complete its trip.

This operation generates an important increase in cost for the trip. No collaboration is possible between vehicles because real-time monitoring system based on GPS and radio communication between vehicles are not widely distributed due to supplementary investments.

Let us consider a CARP with only 9 arcs and 3 vehicles. Assume the solution cost is 100 and composed of 3 trips (figure 1). This solution x has a deterministic cost $h(x)=100$. Assume that the vehicle capacity is 4 and that the demand per arc is 1 unit. The total loads of vehicles 1, 2 and 3 are respectively 3 (serviced arcs 7,8,9), 2 (serviced arcs 1,2) and 4 (serviced arcs 3,4,5,6).

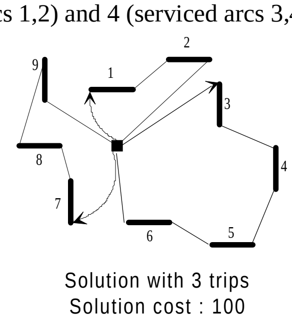

Figure 1: Solution of the Deterministic CARP

In practice, the quantities collected may differ from estimated values. Assume that after the service of arc 4, the total vehicle load is 3.5 and that the demand is 1.2 and 1.4 for arcs 5 and 6.

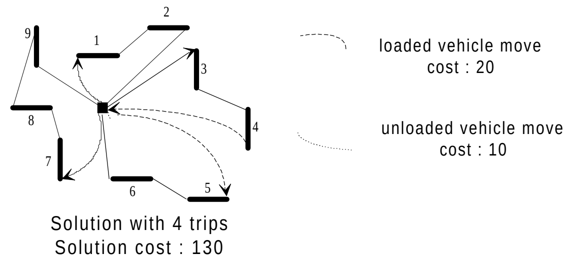

Figure 2: A solution of the Deterministic CARP with random events

Because the vehicle cannot service arc 5, it moves from its current position (end of arc 4) to the depot node and moves from the depot node to the begin of the arc 5 to finish the trip initially planned (figure 2). Due to this unproductive move, the solution cost becomes $H(x,\omega)=130$.

2.3 Modelling of random events

Because of the large number of bins to collect in each street, the central limit theorem shows that the quantities variations can be efficiently modelled by a Gauss distribution $N(r_{ij},\sigma_{ij}^2)$. The Gaussian distribution is truncated to avoid values less than 0 and greater than the vehicle capacity. Note that these events are anyway unlikely. Figure 3 provides an outline of the algorithm used to model variations in quantities. The standard deviation parameter is assigned $r_{ij}/10$ in urban waste collection for example. This model does not take into account special public holiday that increase waste quantities simultaneously.

```
    For all serviced arcs Do
        q ← N(r_ij, Dev)
        If q > Q Then
// no quantity greater than vehicle capacity
            q ← Q
        End If
        If q ≤ 0 Then
// no negative or null quantity for serviced arc
            q ← 1
        End If
    End Do
```

Figure 3: Algorithm used to model quantity variations

3 GENERAL FRAMEWORK

At first, an optimisation process is carried out to obtain a solution of the Deterministic CARP. Depending on the criteria to minimize and on the method used, the solutions are optimal or quasi-optimal. Secondly, replications are provided using a stochastic model of the network and further statistical information are computed: average cost, standard deviation and so on (figure 4). The replications are used to evaluate the deterministic solution in stochastic case. One replication uses randomly distributed quantities that simulate random events in CARP and allows to evaluate $H(x,\omega)$.

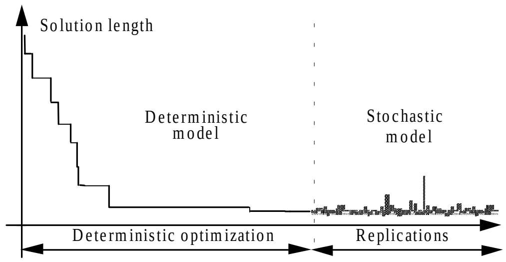

Figure 4: Framework

3.1 Sensitivity analysis of solutions (DCARP)

The HGA is used to build the deterministic solutions. It outperforms the previously best algorithm, the CARPET tabu search method (Hertz *et al.*, 2000). The quasi-optimal solutions reported using such an approach are solutions for the Deterministic CARP. In most trips, the vehicles tend to be full or nearly full.

3.2 Increasing sensitivity of the solutions

To avoid solutions in which trip loads are close to vehicle capacity, we propose to reduce vehicle capacity of p percents before the optimisation process. The trips implying unprofitable moves to the depot node are those for which the total quantity collected is very close to the vehicle capacity and then must be avoided.

Such an approach induces solutions with a higher number of trips and increases the solution cost. From a deterministic point of view, the solutions are less profitable than the first one. During the replications process, the vehicle capacity remains equal to the initial vehicle capacity and statistical information on solutions robustness are collected. The solutions computed with such an approach are efficient solutions of the Stochastic CARP because they have some promising characteristics of robustness. To obtain a readable presentation we denote ‘Tight Approach’ the first proposed approach and ‘Slack Approach’ the second one.

3.3 Replications

During the replications the following criteria are computed:

- empirical average cost;
- empirical average number of trips;
- empirical standard deviation of the cost;
- empirical standard deviation of the number of trips;
- percentage of replications reporting a higher number of trips compared to the HGA solution.

Both approaches are compared using the average and the standard deviation of the relevant criteria but the percentage of replications reporting a higher number of trips is the most relevant criterion in practice..

4 EXPERIMENTS

In order to obtain reliable and sound results, the experiments have been carried out using the well-known benchmarks proposed by Belenguer, Eglese and DeArmon. We only present in this paper the experiments for DeArmon’s instances (table 1). Similar results have been obtained using Belenguer' s and Eglese’s instances. The experiments have been on a Pentium III using Windows 98 operating system. The program has been developed using Delphi 5.0 package. 1,000 replications have been performed and the vehicle capacity has been reduced by 10% during the optimisation process for the Slack Approach. At the end of the deterministic optimisation process the following values are reported:

- cost of the best solution found,
- number of trips in the best solution found,

5 COMPARISON OF THE ROBUSTNESS FOR THE TWO APPROACHES ON DEARMON'S INSTANCES

5.1 Some remarks on the results of the Tight Approach

For the 23 instances (table 1), the solution cost increases by about 8% during the replications and the number of trips required increases by about 23%. On average the percentage of iterations reporting supplementary trips is about 70%. For all instances, the solutions report an enlarged number of trips and a significant increase in total cost.

For example, the cost of the solution found at the end of the deterministic optimisation process for the instance Gdb1 is 316 and this value is optimal. The average cost over 1,000 replications is 351.23 and the standard deviation is 31.17. So, the impact of quantities variations on the solution cost cannot be neglected. In the illustrative instance 1, 66.20% of replications report a supplementary number of trips

Problem	Cost	Number of trips	Average cost	Average number of trips	Percent of iterations with supplementary number of trips	Solution cost: standard deviation	Number of trips: standard deviation
gdb1	316	5	351.23	5.82	66.20	31.17	0.68
gdb2	339	6	389.50	8.08	94.20	28.06	1.10
gdb3	275	5	292.82	5.81	64.40	15.34	0.70
gdb4	287	4	320.93	5.24	79.70	23.58	0.86
gdb5	377	6	443.17	8.09	92.70	37.67	1.12
gdb6	298	5	335.00	6.25	79.40	27.98	0.86
gdb7	325	5	354.67	5.82	65.00	25.27	0.70
gdb8	350	10	385.10	11.99	92.30	21.19	1.09
gdb9	303	10	362.98	13.58	99.30	24.58	1.40
gdb10	275	4	286.29	4.43	43.00	13.12	0.50
gdb11	395	5	412.21	5.67	58.90	15.63	0.62
gdb12	458	7	511.76	8.18	78.20	39.93	0.85
gdb13	536	6	584.81	8.81	97.60	24.71	1.21
gdb14	100	5	102.45	5.47	40.30	3.74	0.63
gdb15	58	4	58.06	4.03	2.90	0.34	0.17
gdb16	127	5	134.46	6.63	88.50	5.23	0.94
gdb17	91	6	91.22	6.11	11.10	0.63	0.31
gdb18	164	5	167.02	5.30	28.70	4.91	0.49
gdb19	55	3	60.50	3.69	59.20	5.78	0.64
gdb20	121	4	129.40	5.74	91.00	5.39	0.95
gdb21	156	6	164.59	7.69	90.20	5.37	0.95
gdb22	200	8	205.82	9.64	87.90	3.63	1.02
gdb23	233	10	248.73	13.79	99.30	6.69	1.50

Table 1: Sensitivity of solutions with the Tight Approach

5.2 Some remarks on the results of the Slack Approach

For the 23 instances (table 2), the solution cost increases only by about 0.06 % during replications and the number of trips remains nearly constant: it increases by about 0.11%. On average the percent of iteration reporting supplementary trips is about 1.05%.

For the 23 instances, compared with the Tight Approach, one can see that:

- the solution cost found at the end of the deterministic optimisation process is higher with the Slack Approach;
- the number of trips required is also greater;
- the average solution cost in the replications is smaller;
- the average number of trips is also reduced;
- the percentage of replications reporting extra trips is lower.

For the Gdb1 instance for example, the solution cost is 337 with the Slack Approach. Remember that the HGA cost was 316 using the Tight Approach. The number of trips required becomes 6: one extra trip has been added on top of the 5 trips computed with the Tight Approach.

The average solution cost after replications is also 337 (to be compared to the previous average 351.23) and the number of trips is still 6. Therefore, the HGA solution computed without filling vehicles completely is very robust, since it is not affected by replications.

Although the solutions with the Slack are less promising at the end of the HGA, the stochastic evaluations prove that these solutions are more useful from an industrial point of view, in avoiding excessive waste of time when facing random quantities variations. Since real life problems are stochastic, the Slack Approach is more promising.

Problem	Cost	Number of trips	Average cost	Average number of trips	Percent of iterations with supplementary number of trips	Solution cost: standard deviation	Number of trips: standard deviation
gdb1	337	6	337.00	6.00	0.00	0.00	0.00
gdb2	359	7	359.00	7.00	0.00	0.00	0.00
gdb3	296	6	296.00	6.00	0.00	0.00	0.00
gdb4	313	5	313.00	5.00	0.00	0.00	0.00
gdb5	409	7	409.00	7.00	0.00	0.00	0.00
gdb6	324	6	324.00	6.00	0.00	0.00	0.00
gdb7	351	6	351.00	6.00	0.00	0.00	0.00
gdb8	370	11	371.65	11.07	6.80	6.34	0.25
gdb9	331	12	331.93	12.05	4.70	4.58	0.22
gdb10	283	5	283.11	5.01	0.70	1.32	0.08
gdb11	403	6	403.00	6.00	0.00	0.00	0.00
gdb12	478	8	480.21	8.04	4.00	12.13	0.20
gdb13	544	7	544.23	7.01	1.00	2.60	0.10
gdb14	100	5	100.05	5.00	0.50	0.71	0.07
gdb15	58	4	58.00	4.00	0.10	0.06	0.03
gdb16	129	6	129.02	6.01	0.70	0.30	0.08
gdb17	91	6	91.00	6.00	0.00	0.00	0.00
gdb18	164	5	164.03	5.00	0.40	0.50	0.06
gdb19	63	4	63.00	4.00	0.00	0.00	0.00
gdb20	123	5	123.05	5.00	0.50	0.71	0.07
gdb21	158	7	158.05	7.01	1.20	0.44	0.11
gdb22	202	9	202.10	9.03	2.70	0.61	0.17
gdb23	237	12	237.03	12.01	1.00	0.27	0.10

Table 2: Sensitivity of solutions with the Slack Approach

5.3 Final comments on the computation results with the Tight and Slack Approaches

Table 3 compares the relative performances of the two approaches. Note that at the end of the optimisation process compared to the Tight Approach:

- the solution cost increases by about 4.43% on average with the Slack Approach;
- the number of trips increases by 16.16% on average with the Slack Approach.

The stochastic evaluation of the solutions proves:

- the average cost is less that 3.20% using the Slack Approach;
- the average number of trips is less that 4.65% using the Slack Approach.

The percentage of replications reporting supplementary trips has been reduced from 70% on average.

One of the most relevant criteria is the percentage of replications with extra trips. Such supplementary trips must be avoided because they immediately induce an increase in solution cost. Figure 5 and table 3 show there is no doubt about the efficiency of the Slack Approach compared to the Tight one.

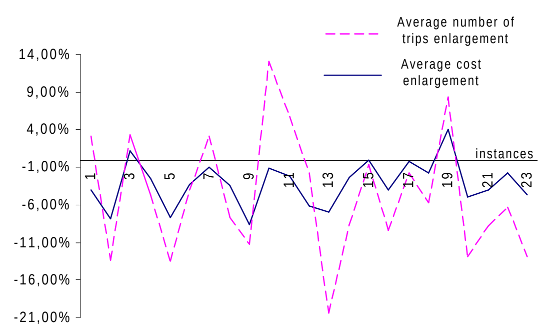

Figure 5: Comparison based on stochastic evaluation

6 SUMMARY

This paper presents the first evaluation of sensitivity of CARP solutions and the first approach to obtain high-quality solutions as regards the robustness required from an industrial point of view.

The best-known solutions ever published for deterministic CARP instances are evaluated regarding quantities variations. To reach this goal, statistics are gathered by replications. The results prove that the solutions are unprofitable from an industrial point of view due to the extra cost induced by unforeseen moves to the depot node.

A new approach, the Slack one, is proposed to obtain trips for which the quantities collected are not close to vehicle capacity. It permits to protect the HGA solutions against quantity fluctuations. This approach computes high- quality solutions in terms of robustness. Compared to the Tight Approach:

- the cost evaluation over 1,000 replications is improved;
- the number of trips evaluation over 1,000 replications is diminished;
- the percentage of replications reporting additional moves to the depot node is highly reduced.

This work has multiple future useful extensions. Further research is now directed:

- to a mathematical analysis permitting to evaluate the robustness of solution during the HGA iterations (so, without the help of replications),
- to the study of the ECARP (Extended CARP) for which random events on quantities to collect imply variations in service cost (Lacomme *et al.*, 2001b).

	Comparison at the end of the Deterministic process optimization		Comparison based on stochastic evaluation		
Problem	Cost enlargement	Percent of supplementary trips	Average cost enlargement	Average number of trips enlargement	Improvement of the percent of the iterations reporting supplementary trips
gdb1	6.65 %	20.00 %	**-4.05 %**	3.09 %	100;00 %
gdb2	5.90 %	16.67 %	**-7.83 %**	**-13.37 %**	100;00 %
gdb3	7.64 %	20.00 %	1.09 %	3.27 %	100;00 %
gdb4	9.06 %	25.00 %	**-2.47 %**	**-4.58 %**	100;00 %
gdb5	8.49 %	16.67 %	**-7.71 %**	**-13.47 %**	100;00 %
gdb6	8.72 %	20.00 %	**-3.28 %**	**-4.00 %**	100;00 %
gdb7	8.00 %	20.00 %	**-1.03 %**	3.09 %	100;00 %
gdb8	5.71 %	10.00 %	**-3.49 %**	**-7.67 %**	92.63 %
gdb9	9.24 %	20.00 %	**-8.55 %**	**-11.27 %**	95.27 %
gdb10	2.91 %	25.00 %	**-1.11 %**	13.09 %	98.37 %
gdb11	2.03 %	20.00 %	**-2.23 %**	5.82 %	100.00 %
gdb12	4.37 %	14.29 %	**-6.16 %**	**-1.71 %**	94.88 %
gdb13	1.49 %	16.67 %	**-6.94 %**	**-20.43 %**	98.98 %
gdb14	0.00 %	0.00 %	**-2.34 %**	**-8.59 %**	98.76 %
gdb15	0.00 %	0.00 %	**-0.10 %**	**-0.74 %**	96.55 %
gdb16	1.57 %	20.00 %	**-4.05 %**	**-9.35 %**	99.21 %
gdb17	0.00 %	0.00 %	**-0.24 %**	**-1.80 %**	100.00 %
gdb18	0.00 %	0.00 %	**-1.79 %**	**-5.66 %**	98.61 %
gdb19	14.55 %	33.33 %	4.13 %	8.40 %	100.00 %
gdb20	1.65 %	25.00 %	**-4.91 %**	**-12.89 %**	99.45 %
gdb21	1.28 %	16.66 %	**-3.97 %**	**-8.84 %**	98.67 %
gdb22	1.00 %	12.5 %	**-1.81 %**	**-6.33 %**	96.93 %
gdb23	1.72 %	20.00 %	**-4.70 %**	**-12.91 %**	98.99 %
Average:	**4.43 %**	**16.16 %**	**-3.20 %**	**-4.65 %**	**98.58 %**

Table 3: Relative performances of both approaches for DeArmon's instances

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