AN INTEGER LINEAR MODEL FOR GENERAL ARC ROUTING PROBLEMS

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ABSTRACT

This research focuses on an extended version of the Capacitated Arc Routing Problem (CARP), obtained by adding several realistic constraints to the basic problem. We propose an integer linear model for this extended CARP (E-CARP) and an implementation in the Xpress modelling language. A set of 11 benchmark problems is built to evaluate the practical usefulness of the new model. These examples result from modifications of some CARP instances available in the literature. For small and middle scale instances the model allows to compute optimal solutions or high-quality solutions in acceptable computational time limits. The E-CARP is a step towards more realistic arc routing models, like the ones occurring in municipal waste collection.

INTRODUCTION

The basic *Capacitated Arc Routing Problem* (*CARP*) is defined in the literature on an undirected network $G = (V,E)$ with a set V of n nodes and a set E of m edges. A fleet of identical vehicles of capacity Q is based at a depot node s. A subset R of *required edges* require service by a vehicle. All edges can be traversed any number of times. Each edge (i,j) has a traversal cost $c_{ij} \geq 0$ and a demand $r_{ij} \geq 0$.

The CARP consists of determining a set of vehicle trips of minimum total cost, such that each trip starts and ends at the depot, each required edge is serviced by one single trip, and the total demand handled by any vehicle does not exceed Q. The cost of a trip comprises the costs of its serviced edges and of its intermediate connecting paths. Many applications occur in road networks: urban waste collection, snow plowing, sweeping, gritting, etc. Demands are usually amounts to be collected along the streets (urban waste) or delivered (salt in winter). Costs are often distances or travel times.

The undirected version concern roads whose both sides can be serviced during one traversal and in any direction, a common situation in low-traffic suburban areas. A directed version is sometimes studied: each arc represents one street (or one side of street) with an imposed direction for service. Note the difference between an edge (i,j) and a pair of opposite arcs (i,j) and (j,i): both represent a bi-directional street but each side need be serviced separately for the pair of arcs. Both the directed and undirected CARP are NP-hard, even in the single-vehicle case (*Rural Postman Problem*).

In applications like municipal weight collection, the street network is obviously *mixed* (with edges *and* arcs) and some turns are not allowed. Moreover, the time to drive thru a street without collecting it is smaller than its collecting time. The E-CARP denotes an extended CARP for tackling such applications:

- the network is mixed
- each link (arc or edge) has a true traversal cost c_{ij} (without service) distinct from its servicing cost w_{ij}
- prohibited turns.

INTEGER LINEAR FORMULATION

Our integer linear formulation generalizes a model proposed by Golden and Wong in 1981 for the basic undirected CARP (Golden and Wong 1981). Note that even this very first model has never been evaluated, due to the lack of solvers powerful enough at that time.

Input data

The mixed network is coded as a directed multigraph $G = (V,A)$. V is a set of n nodes with a depot at node 1. A is a set of m arcs in which each edge is coded as two opposite arcs, both inheriting the costs and demand of the edge. Since we allow several arcs from a node i to a node j, we use arc indexes from 1 to m instead of pairs like (i,j). This avoids ambiguities and makes the model more concise. Each arc u has one begin node $b(u)$, one end node $e(u)$, a traversal cost (without service) c_u and a demand q_u. Like in waste collection, we assume that the set R of the r required arcs contains all arcs with non-zero demands. Such arcs have also a servicing cost w_u. A fleet of K identical vehicles of capacity Q is based at the depot. All costs, demands and capacities are non-negative real numbers. No demand exceeds Q, *i.e.* $Q \geq \max \{q_u \mid u \in A\}$.

For tackling the mixed graph and prohibited turns, let us note for each arc $u = (i,j)$:

- $S(u)$ the set of adjacent arcs $v = (j,k)$ that a vehicle may traverse immediately after the traversal of u,
- $P(u)$ the set of adjacent arcs $v = (k,i)$ that a vehicle may traverse immediately before traversing u,
- $Inv(u)$ the index of the opposite arc.

$S(u)$ and $P(u)$ can describe many types of constrained turns, imposed by a road sign, an excessive turning circle, a too narrow street, etc. All these cases are handled by excluding v from $S(u)$ and u from $P(v)$. $Inv(u)$ is required to distinguish between a genuine arc and a pair of arcs coding an edge. If two arcs u and v represent the same edge, then $Inv(u) = v$ and $Inv(v) = u$. If u and v are two opposite arcs requiring service separately, then $Inv(u) = Inv(v) = 0$.

Decisional variables

- $x_{u,v}^{p}$ number of times vehicle p uses arc v after arc u
- $l_{u}^{p} = 1$ if arc u is serviced by vehicle p, 0 otherwise.

Note that the initial model proposed by Golden and Wong uses binary variables x_{up} equal to 1 if and only if vehicle p traverses arc u. These variables are binary thanks to a theorem for the undirected CARP, which states that *there exists an optimal solution in which no trip traverses the same edge more than once, in a given direction*. This property is no longer valid for directed or mixed graphs and the variables become integral.

Moreover, the x variables are now indexed by two arcs to handle prohibited turns correctly. They need be defined only for all permitted pairs of consecutive arcs (u,v). In real road networks, each arc (street) is followed by 4 arcs on average (including a possible U-turn). There are then $4m$ permitted pairs on average, even less in case of prohibited turns. The variables l need be defined only for the r required arcs. The total number of variables is then $K.(4m+r)$ in practice.

The E-CARP model

$$(1) \quad \text{Min} \sum_{p=1}^{K} \sum_{v \in R} w_v \times l_v^p + \sum_{p=1}^{K} \sum_{v \in R} \sum_{u \in P(v)} c_v \times \left(x_{u,v}^p - l_v^p \right) + \sum_{p=1}^{K} \sum_{v \in A-R} \sum_{u \in P(v)} c_v \times x_{u,v}^p$$

subject to:

$$(2) \quad \forall p = 1 \ldots K, \forall u \in A: \sum_{v \in P(u)} x_{v,u}^p = \sum_{v \in S(u)} x_{u,v}^p$$

$$(3) \quad \forall u \in R, Inv(u) = 0: \sum_{p=1}^{K} l_u^p = 1$$

$$(4) \quad \forall u \in R, Inv(u) > 0: \sum_{p=1}^{K} \left(l_u^p + l_{Inv(u)}^p \right) = 1$$

$$(5) \quad \forall p = 1 \ldots K, \forall u \in R: \sum_{v \in S(u)} x_{u,v}^p \geq l_u^p$$

$$(6) \quad \forall p = 1 \ldots K: \sum_{u \in R} l_u^p \times q_u \leq Q$$

$$(7) \quad \forall p = 1 \ldots K, \forall i = 2 \ldots n: \sum_{\substack{u \in A \\ b(u)=i}} f_u^p - \sum_{\substack{u \in A \\ e(u)=i}} f_u^p = \sum_{\substack{u \in R \\ b(u)=i}} l_u^p$$

$$(8) \quad \forall p = 1 \ldots K, \forall u \in A: f_u^p \leq n^2 \times \sum_{v \in S(u)} x_{u,v}^p$$

$$(9) \quad \forall p = 1 \ldots K, \forall u \in A, \forall v \in S(u): x_{u,v}^p \in \mathbb{N}$$

$$(10) \quad \forall p = 1 \ldots K, \forall u \in R: l_u^p \in \{0,1\}$$

$$(11) \quad \forall p = 1..K, \forall u \in A: f_u^p \geq 0$$

The three main terms in the objective function (1) correspond to the total servicing cost of the required arcs, the total traversal cost of the required arcs, and the total traversal cost of non-required arcs. In each flow constraint (2), the number of times vehicle k enters arc u must be equal to the number of times it leaves u. In the example of figure 1, the values on each arc are the number of traversals by vehicle k. We can check that arc $u = (1,2)$ is entered and left four times.

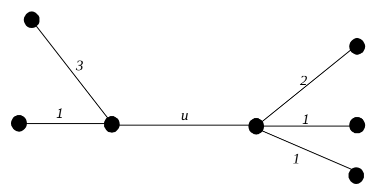

Figure 1: Flow constraints on arc u.

Constraints (3) mean that each required true arc (*i.e.*, not an edge) of the original network must be collected, and by one single vehicle. Constraints (4) concern required edges. When two arcs u and $Inv(u)$ code the same edge, only one of them must be collected, and by one single vehicle. The constraints (5) prevent the required arcs from being serviced by a vehicle when the vehicle does not use them.

Constraints (6) indicate that the total among of demand serviced by one vehicle cannot exceed its capacity. Constraints (7) and (8) generalize subtours elimination constraints proposed by Golden and Wong. The total number of constraints (without positivity and bound constraints) is $K(2m+n+1)+r(K+1)$. For street networks $m \approx 4n$ and we obtain $K(9n+1)+r(K+1)$ constraints.

A LIBRARY FOR THE E-CARP

For the basic undirected CARP, two sets of instances have been proposed by DeArmon (DeArmon 1981) and by Belenguer and Benavent (Belenguer and Benavent 1997). All edges in these instances require service. The data files can be downloaded via the Internet at address: *ftp://matheron.estadi.uv.es/pub/CARP*. Since no such benchmarks are available for our E-CARP, we propose to use some DeArmon's instances and to convert them into E-CARP instances by adding new constraints.

A library composed of 11 instances based on DeArmon's files gdb1, gdb2, gdb3, gdb4, gdb5, gdb14, gdb15, gdb16, gdb17, gdb18, gdb19 is proposed. The modified files have a name ending with "e" *e.g.*, gdb1 gives the E-CARP instance gdb1e. They can be obtained by sending a request by e-mail to the authors. The instances are solved using the Xpress linear programming software (*http://www.dash.co.uk*), on a 600 MHz PC under Windows 95. We detail hereafter the solutions obtained for problems gdb15e and gdb19e, whose data are listed in table 1.

Table 1: Definition of 2 problems of our E-CARP Library

Problem name	Arcs reachable after one arc : u : $S(u)$	c_u , w_u , q_u
Gdb15e	1 : 7, 8, 9,10,11,22	1 , 1, 2
	2 : 12,13,14,15,23,28	1 , 1, 3
Number of nodes: 7	3 : 16,17,18,24,29,33	1 , 1, 4
Number of arcs: 42	4 : 19,20,25,30,34,37	1 , 1, 5
Vehicle capacity: 37	5 : 21,26,31,35,38,40	1 , 1, 6
Number of vehicles: 3	6 : 27,32,36,39,41,42	1 , 1, 7
	7 : 12,13,14,15,23,28	2 , 2, 3
Only arcs with $q_u > 0$ are	8 : 16,17,18,24,29,33	2 , 2, 4
reported below.	9 : 19,20,25,30,34,37	2 , 2, 0
$\forall i \notin \{1,22\}\ Inv(i) = i + 21$	10: 21,26,31,35,38,40	2 , 2, 0
$Inv(1) = 0$	11: 27,32,36,39,41,42	2 , 2, 0
$Inv(22) = 0$	12: 16,17,18,24,29,33	3 , 3, 4
	13: 19,20,25,30,34,37	3 , 3, 5
	14: 21,26,31,35,38,40	3 , 3, 0
	15: 27,32,36,39,41,42	3 , 3, 0
	16: 19,20,25,30,34,37	4 , 4, 5
	17: 21,26,31,35,38,40	4 , 4, 6
	18: 27,32,36,39,41,42	4 , 4, 0
	19: 21,26,31,35,38,40	5 , 5, 6
	20: 27,32,36,39,41,42	5 , 5, 7
	21: 27,32,36,39,41,42	6 , 6, 7
	22: 1, 6, 0, 0, 0, 0	1 , 1, 2
	23: 1, 2, 3, 4, 5, 6	1 , 1, 3
	24: 1, 2, 3, 4, 5, 6	1 , 1, 4
	25: 1, 2, 3, 4, 5, 6	1 , 1, 5
	26: 1, 2, 3, 4, 5, 6	1 , 1, 6
	27: 1, 2, 3, 4, 5, 6	1 , 1, 7
	28: 7, 8, 9,10,11,22	2 , 2, 3
	29: 7, 8, 9,10,11,22	2 , 2, 4
	30: 7, 8, 9,10,11,22	2 , 2, 0
	31: 7, 8, 9,10,11,22	2 , 2, 0
	32: 7, 8, 9,10,11,22	2 , 2, 0
	33: 12,13,14,15,23,28	3 , 3, 4
	34: 12,13,14,15,23,28	3 , 3, 5
	35: 12,13,14,15,23,28	3 , 3, 0
	36: 12,13,14,15,23,28	3 , 3, 0
	37: 16,17,18,24,29,33	4 , 4, 5
	38: 16,17,18,24,29,33	4 , 4, 6
	39: 16,17,18,24,29,33	4 , 4, 0
	40: 19,20,25,30,34,37	5 , 5, 6
	41: 19,20,25,30,34,37	5 , 5, 7
	42: 21,26,31,35,38,40	6 ,12, 7
Gdb19e	1 : 6, 7, 8, 5, 12,0	4, 5, 0
Number of nodes: 8	2 : 17,13, 0, 0, 0, 0	3, 4, 3
Number of arcs: 44	3 : 14, 9, 0, 0, 0, 0	1, 2, 5
Vehicle capacity: 27	4 : 15,10, 0, 0, 0, 0	2, 8, 8
Number of vehicles: 3	5 : 22,16, 0, 0, 0, 0	1, 2, 4
	6 : 17,13, 0, 0, 0, 0	9,10, 6
Only arcs with $q_u > 0$ are	7 : 14, 9,18, 0, 0, 0	5, 6, 1
reported below.	8 : 19,20,11, 0, 0, 0	2, 3, 9
$\forall i \notin \{11,12,22\}\ Inv(i) = i + 11$	9 : 20,19,11, 0, 0, 0	7, 8, 0
$Inv(11) = 0$	10: 21, 0, 0, 0, 0, 0	5, 6, 5
$Inv(12) = 0$	11: 22,16, 0, 0, 0, 0	6, 7, 8
$Inv(22) = 0$	12: 1, 2, 3, 4, 0, 0	4, 5, 8
	13: 1, 2, 3, 4, 0, 0	3, 4, 3
	14: 1, 2, 3, 4, 0, 0	1, 2, 5
	15: 1, 2, 3, 4, 0, 0	2, 3, 8
	16: 12, 6, 7, 8, 5, 0	1, 2, 4
	17: 6,12, 7, 8, 5, 0	9,10, 6
	18: 7,12, 6, 8, 5, 0	5, 6, 1
	19: 8, 5, 7, 6,12, 0	2, 3, 9
	20: 9,18,14, 0, 0, 0	7, 8, 0
	21: 10,15, 0, 0, 0, 0	5, 6, 5
	22: 11,20, 0, 0, 0, 0	6, 7, 8

SOLUTIONS TO SOME E-CARP INSTANCES

The gdb19e instance

The gdb19e instance has been obtained by adding the following constraints to DeArmon's gdb19 file :

- prohibited turn added from arc 3 to arc 18,
- prohibited turn added from arc 22 to arc 19,
- no demand on arcs 9 = (5,7), 20 = (7,5) and 1 = (1,2),
- arcs 22 = (3,7) and 11 = (7,3) become independent, *i.e.* *Inv*(22) = *Inv*(11) = 0. The edge (3,7) is replaced by arcs 11 and 22 and these arcs must be serviced separately.
- the edge (1,2) is replaced by arcs 1 and 12 with only arc 12 to be serviced.

Figure 2 shows a graphical representation of the graph, corresponding to the data listed in table 1. The four numbers written as *a*(*b*,*c*,*d*) on each arc are the arc number *a*, the traversal cost *b*, the servicing cost *c*, and the demand *c*. A vehicle capacity of 27 is assumed.

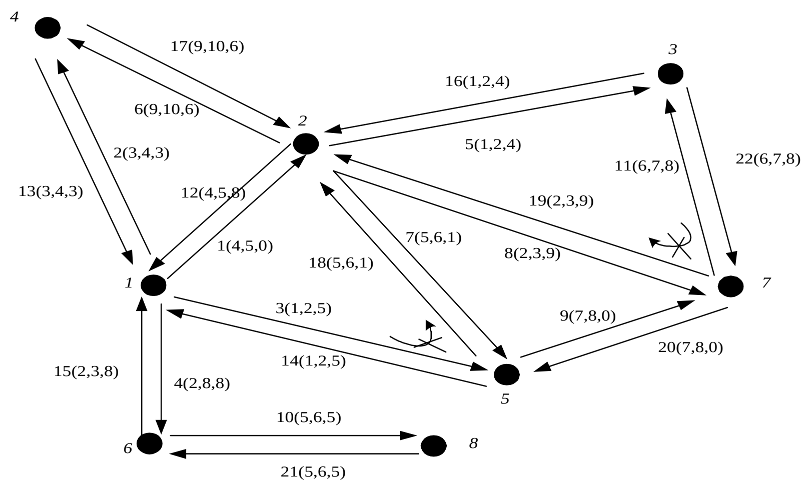

Figure 2: The gdb19e graph

The optimal solution computed using the Xpress model has a total cost equal to 69. Here is an example of Xpress solution listing, in which the *X* variables represent the trip of each vehicle. Each variable *X* has three indices and $X\_x_1\, x_2\, x_3$ can be read as follows:

- x_3 is the vehicle number,
- x_2 is the arc traversed by vehicle x_3,
- x_1 is the arc used before x_2.

The trips:

```
Vehicle 1
"X__12021",    1.000000
"X__19081",    1.000000
"X__17121",    1.000000
"X__02171",    1.000000
"X__08191",    1.000000

Vehicle 2
"X__15042",    1.000000
"X__04102",    1.000000
"X__21152",    1.000000
"X__10212",    1.000000

Vehicle 3
"X__14013",    1.000000
"X__16053",    1.000000
"X__01073",    1.000000
"X__22113",    1.000000
"X__07143",    1.000000
"X__05163",    1.000000
"X__11223",    1.000000
```

The serviced arcs :

```
Vehicle 1
"L__12021",    1.000000
"L__19081",    1.000000
"L__17121",    1.000000
"L__02171",    1.000000

Vehicle 2
"L__21152",    1.000000
"L__10212",    1.000000

Vehicle 3
"L__01073",    1.000000
"L__22113",    1.000000
"L__07143",    1.000000
"L__05163",    1.000000
"L__11223",    1.000000
```

For the serviced arcs a similar notation is used. The corresponding trips are graphically depicted in figure 3.

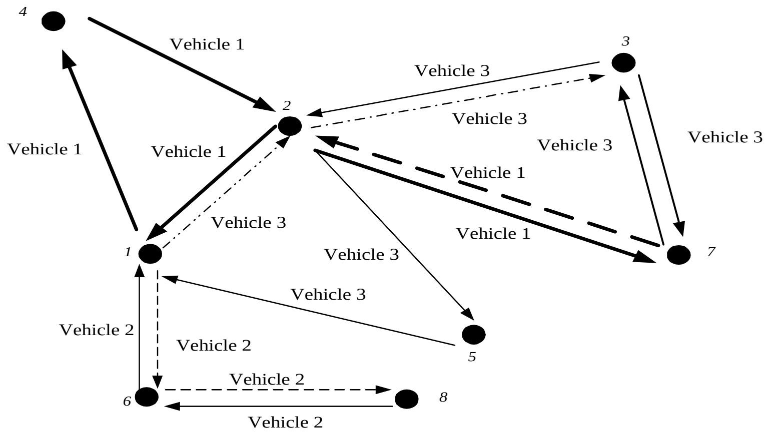

Figure 3: Optimal solution for gdb19e

The gdb15e instance

The computation of the optimal solution is here too time consuming and only a solution of cost 67 is available, as described by figure 4.

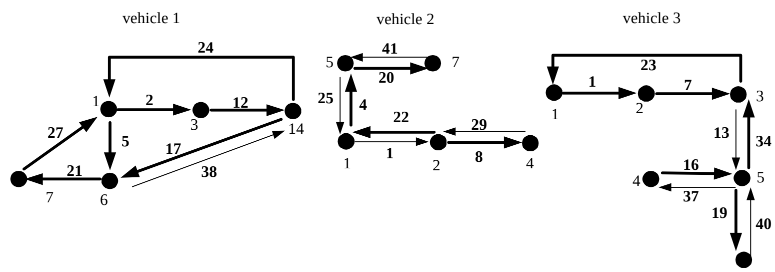

Figure 4: The Gdb15d best known solution

Best-known solutions for the E-CARP library

Table 2: Optimal and best-known solutions

Name	Vertices/arcs	Best-known solution	Optimal
Gdb1e	12x44	347	
Gdb2e	12x52	406	
Gdb3e	12x44	298	
Gdb4e	**11x38**	**297**	**Yes**
Gdb5e	16x52	485	
Gdb14e	**7x42**	**135**	**Yes**
Gdb15e	7x42	67	
Gdb16e	8x56	145	
Gdb17e	8x56	91	
Gdb18e	9x72	152	
Gdb19e	**8x22**	**69**	**Yes**

FINAL COMMENTS

This paper presents the first formulation for an extended CARP. A library for this E-CARP problem is proposed with 11 instances including prohibited turns and a mixed network with arcs and edges. The integer linear formulation of the E-CARP allows to solve optimally some small and medium-size instances. This approach permits to take into account many constraints of municipal waste collection for example.

This work has multiple future useful extensions. Further research could be directed:

- to the development of powerful lower bounds based on the previous work of Belenguer and Benavent for example,
- to the development of heuristics to solve the linear program.
- to couple this exact approach to metaheuristics. This approach could permit to solve optimally some parts of the graph. This topic is investigated with the GA proposed by (Lacomme *et al*. 2001)

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