COMPUTATION OF LINEAR RANK-WIDTH

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Introduction. Rank-width is a complexity measure introduced by Oum and Seymour [7, 6]. Rank-width is interesting for several reasons.

(1) It is equivalent to clique-width, a complexity measure introduced by Courcelle et al. [4], that generalises the well-known complexity measure tree-width introduced by Robertson and Seymour in their graph minors series.

(2) It is algorithmically more interesting than clique-width because we can recognise in polynomial time graphs of rank-width at most $k$ (for fixed $k$)

(3) It shares with tree-width many structural properties (its is for instance related to the theory of matroids and is related to the vertex-minor relation [6]).

(4) . . .

While there exist several algorithms for computing the tree-width of some graph classes, a little is known about the computation of the rank-width of some graph classes. In particular, it is open whether one can compute the rank-width of circle graphs in polynomial time (circle graphs are graphs that are conjectured to play a central role in vertex-minor theory, a role similar to the one of planar graphs in the graph minor theory). A circle graph is an intersection graph of chords in a circle.

In this internship, we are interested in a linearised version of rank-width, called linear rank-width (see for instance [5] for a definition). Indeed, the decomposition associated to rank-width is a tree, and for linear rank-width we impose that tree to be a caterpillar. Linear rank-width is to rank-width what path-width is to tree-width.

The goal. In [2] the authors have given a linear time algorithm to compute the linear rank-width and the linear clique-width of trees (a subclass of circle graphs). The goal of the internship is to implement both algorithms and to adapt them to the case of linear Boolean-width (Boolean-width is a complexity measure similar to rank-width, but the measure is with respect to Boolean algebra instead of $F_2$; a definition can be found in [3]). If there is time we will look at the generation of trees that are obstructions to linear rank-width.
All materials and needed explanations will be provided and for further information please feel free to contact me at kante@isima.fr. You can find the definition of rank-width and clique-width in my Phd Thesis available online [8, Chapter 1]. The papers in the references are available in the authors’ webpages. An implementation of an algorithm for approximating the rank-width of graphs of rank-width at most \( k \) (for fixed \( k \)) exists in SAGE [1].

References